Solution : As you know, $q(t)$ will be a multiple of $2t + 1$ only, if $2t + 1$ divides $q(t)$ leaving remainder zero. Now, taking $2t + 1 = 0$, we have $t = -\frac{1}{2}\cdot$

Also, $$q\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 + 4\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1 = -\frac{1}{2} + 1 + \frac{1}{2} - 1 = 0$$

So the remainder obtained on dividing $q(t)$ by $2t + 1$ is 0.

So, $2t + 1$ is a factor of the given polynomial $q(t)$, that is $q(t)$ is a multiple of $2t + 1$.

EXERCISE 2.3

1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

   (i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) x

   (iv) $x + \pi$ (v) $5 + 2x$

2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.
3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

2.5 Factorisation of Polynomials

Let us now look at the situation of Example 10 above more closely. It tells us that since the remainder, $q\left(-\frac{1}{2}\right) = 0$, $(2t + 1)$ is a factor of $q(t)$, i.e., $q(t) = (2t + 1)\, g(t)$ for some polynomial $g(t)$. This is a particular case of the following theorem.

Factor Theorem : If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then

(i) $x - a$ is a factor of $p(x)$, if $p(a) = 0$, and

(ii) $p(a) = 0$, if $x - a$ is a factor of $p(x)$.

This actually follows immediately from the Remainder Theorem, but we shall not prove it here. However, we shall apply it quite a bit, as in the following examples.

Example 11 : Examine whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$ and of $2x + 4$.

Solution : The zero of $x + 2$ is -2. Let $p(x) = x^3 + 3x^2 + 5x + 6$ and $s(x) = 2x + 4$

Then, $$p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$$